Microwave emission of sonoluminescing bubbles

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Kordomenos *et al.* have attempted to measure single bubble sonoluminescence (SBSL) emission in the microwave window of water in a band of frequencies ranging from 1.65 GHz to 2.35 GHz [Phys. Rev. E **59**, 1781 (1999)]. The sensitivity of the experiment was such that signals greater than 1 nW would have been detected. We show here that this upper bound is compatible with the radiation processes that we think generate significant emission at optical frequencies, electron-neutral and electron-ion bremsstrahlung. In fact, we argue that, almost independently of the specific assumptions concerning the hydrodynamics or the nature of the radiative processes, SBSL intensities exceeding that upper bound can hardly be expected.

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One of the most important research tools for the study of sonoluminescence is spectroscopy. Several calibrated spectra of single bubble sonoluminescence (SBSL) have been recorded in the optical window of water, from 200 to 700 nm wavelength [1]. The analyses of these spectra, or the various efforts to reproduce the spectra from theory, are still not quite conclusive, but we feel that well-known radiative processes are likely responsible for the bulk of the observed continua. The emitting regions of the SBSL environment probably have temperatures between 5000 and $\approx 20\,000$ K, and densities comparable to those of liquids. Emissions from somewhat similar environments (shock waves, capillary discharges) have long been studied in considerable detail [2–4]. Radiative processes, such as electron-atom and electron-ion bremsstrahlung, along with a few others [5,6], are certainly of importance under such conditions. It is noteworthy that the modeling of measured SBSL spectra, based on reliable data related to generation of bremsstrahlung, have been reasonably successful in reproducing the measured spectral profiles and intensities of SBSL [5,7,8].

The only other spectral window of liquid water is in the microwave region. Kordomenos *et al.* have attempted to measure SBSL emission in a band of frequencies ranging from 1.65 GHz to 2.35 GHz [9]. The sensitivity of the experiment was such that signals greater than 1 nW would have been detected. Since no SBSL signals were seen, an upper bound of the emission was thus established. In this paper we want to answer the question whether the SBSL radiation processes that we think generate significant emission at optical frequencies may be expected to generate emission in the microwave region that is consistent with this upper bound [9]. We will show that the signals to be expected at microwave frequencies are many orders of magnitude weaker than that upper bound and could not possibly have been seen in that experiment. In fact, we will argue that, almost independently of the specific assumptions concerning the hydrodynamics or the nature of the radiative processes, SBSL intensities exceeding that upper bound can hardly be expected.

The absorption coefficients of radiative processes, which we think are most relevant for sonoluminescence studies, increase dramatically with decreasing frequencies as we approach the microwave region. For electron-atom bremsstrahlung, this was shown in Fig. 2 of Ref. [10]. For example, in the case of argon, absorption coefficients under SBSL conditions amount to roughly $3\times10^{13}\ \mathrm{m}^{-1}$ at 2 GHz; similar values are obtained for the other rare gases. In the case of electron-ion bremsstrahlung we may use Eq. (21) of Ref. [5] to estimate the absorption coefficient. For a bubble temperature of $T=15\,000$ K, microwave frequency of $\nu=2$ GHz, and gas density of $n_{\mathrm{Gas}}=6\times10^{27}\ \mathrm{m}^{-3}$, we find

$$\kappa_\lambda^{\mathrm{ion}}\approx 5\times10^{14}\ \mathrm{m}^{-1}.$$

These conditions quoted correspond to compression ratios of 1:6 of the equilibrium radius, and a degree of ionization of 1%. With absorption coefficients of that size, the bubble is most certainly optically thick, despite its small size of $R_{\mathrm{min}}<1\ \mu$m. If we add to this simple fact the usual assumption of local thermal equilibrium, we may safely assume that the power emitted by the bubble at these frequencies is given by Planck's blackbody law,

$$P_\nu^{\mathrm{Pl}}d\nu=4\pi R^2\frac{2\pi}{c^2}\frac{h\nu^3}{\exp(h\nu/k_{\mathrm{B}}T)-1}d\nu, \tag{1}$$

regardless of what other radiative processes may be taking place. With this assumption any further treatment of the SBSL environment becomes independent of the nature of the radiative processes present in the bubble. Since $h\nu\ll k_{\mathrm{B}}T$ for any temperature T possibly of interest here we can expand the exponential and write

$$P_\nu^{\mathrm{Pl}}d\nu\approx4\pi R^2\frac{2\pi}{c^2}k_{\mathrm{B}}T\nu^2d\nu, \tag{2}$$

which is easily integrated over the frequency range used in the experiment to yield

$$P^{\mathrm{Pl}}=\alpha R^2T. \tag{3}$$

The constant α equals $3.4\times10^{-11}\ \mathrm{W/(Km^2)}$. Thus with $T=15\,000$ K and $R_{\mathrm{min}}=0.8\ \mu$m yields $P^{\mathrm{Pl}}\approx3.3\times10^{-19}$ W for the peak power emitted by such a bubble. The bubble, however, radiates with this power only for a short time interval of about 100 ps, during which the temperature is large. For the longest part of the acoustic cycle of a length of order

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10 μs no light is emitted. Thus the emission is suppressed further by a factor of about 10^{-5}. We therefore expect a time-averaged emitted power of roughly 10^{-24} W. This is about 15 orders of magnitude less than the upper bound [9]. Even under the most extreme assumptions about the temperatures in the bubble, such as $T=10^8$ K from hypothetical shock wave heating, the emitted intensities would still be orders of magnitude below detection threshold. (Actually, in this case only a small region inside the bubble may be assumed to be that hot [11] since there is not enough kinetic energy in the collapse to heat the whole bubble to such high temperatures, which would only make the probability of an emission exceeding the threshold even more unlikely.)

Moreover, if indeed we have a weakly ionized gas in the bubble, yet another effect becomes important: for an ionization of about 1% at a density of $n_{\mathrm{Gas}}=6\times10^{27}$ m^{-3}, i.e., an electron density of $n_{\mathrm{e}}=6\times10^{25}$ m^{-3}, the plasma frequency [12]

$$\nu_{\mathrm{p}}^2=\frac{n_{\mathrm{e}}e^2}{4\pi^2\epsilon_0 m_{\mathrm{e}}}, \tag{4}$$

amounts to $\nu_{\mathrm{p}}\approx7\times10^{13}$ Hz, which is much greater than the microwave frequencies. Electromagnetic waves of frequencies less than the plasma frequency are attenuated in a plasma roughly according to [12]

$$\kappa_{\mathrm{p}}\approx\frac{4\pi\nu_{\mathrm{p}}}{c}. \tag{5}$$

In our case the attenuation length is thus about $\kappa_{\mathrm{p}}^{-1}\approx0.34$ μm, which is at least of comparable magnitude to the bubble radius, if not smaller. In other words, the opacity of the bubble is even bigger than our bremsstrahlung estimate given above.

We therefore think that just about any theory of light emission of SBSL will at microwave frequencies around 2 GHz be compatible with the experimentally determined upper bound.

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